Laser field absorption in self-generated electron-positron pair plasma

E. N. Nerush, I. Yu. Kostyukov*

Institute of Applied Physics, Russian Academy of Sciences, 603950 Nizhny Novgorod, Russia

A. M. Fedotov, N. B. Narozhny, National Research Nuclear University MEPhI, Moscow, 115409, Russia

N. V. Elkina, H. Ruhl Ludwig-Maximillians Universität München, 80539, Germany

Recently much attention has being attracted to the problem of limitations on the attainable intensity of high power lasers [A.M. Fedotov et al. Phys. Rev. Lett. 105, 080402 (2010)]. The laser energy can be absorbed by electron-positron pair plasma produced from a seed by strong laser field via development of the electromagnetic cascades. The numerical model for self-consistent study of electron-positron pair plasma dynamics is developed. Strong absorption of the laser energy in self-generated overdense electron-positron pair plasma is demonstrated. It is shown that the absorption becomes important for not extremely high laser intensity $I \sim 10^{24} \ {\rm W/cm^2}$ achievable in the nearest future.

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Due to an impressive progress in laser technology, laser pulses with peak intensity of nearly 2×10^{22} W/cm² are now available in the laboratory [1]. When the matter is irradiated by so intense laser pulses ultrarelativistic dense plasma can be produced. Besides of fundamental interest, such plasma is an efficient source of particles and radiation with extreme parameters that opens bright perspectives in development of advanced particle accelerators [2], next generation of radiation sources [3, 4], laboratory modeling of astrophysics phenomena [5], etc. Even higher laser intensities can be achieved with the coming large laser facilities like ELI (Extreme Light Infrastructure) [6] or HiPER (High Power laser Energy Research facility) [7]. At such intensity the radiation reaction and quantum electrodynamics (QED) effects become important [8–13].

One of the QED effects, which has recently attracted much attention, is the electron-positron pair plasma (EPPP) creation in a strong laser field [11, 12]. The plasma can be produced via avalanche-like electromagnetic cascades: the seed charged particles are accelerated in the laser field, then they emit energetic photons, the photons by turn decay in the laser field and create electron-positron pairs. The arising electrons and positrons are accelerated in the laser field and produce new generation of the photons and pairs. It is predicted [12] that an essential part of the laser energy is spent on EPPP production and heating. This can limit the attainable intensity of high power lasers. That prediction was derived using simple estimates, therefore self-consistent treatment based on the first principles is needed.

The collective dynamics of EPPP in strong laser field is a very complex phenomenon and numerical modeling becomes important to explore EPPP. Up to now the numerical models for collective QED effects in strong laser field have been not self-consistent. One approach in numerical modeling is focused on plasma dynamics and neglects the QED processes like pair production in the laser field. It is typically based on particle-in-cell (PIC) methods and uses equation for particle motion with radiation reaction forces taken into account [13]. The second one is based on Monte Carlo (MC) algorithm for photon emission and electron-positron pair production. This approach has been used to study the dynamics of electromagnetic cascades [14]. However, it completely ignores the self-generated fields of EPPP and the reverse effect of EPPP on the external field. The latter effect is especially important to determine the limitations on the intensity of high power lasers [12, 15].

Quantum effects in strong electromagnetic fields can be characterized by the dimensionless invariants [16, 17] $\chi_e = e\hbar/(m^3c^4)|F_{\mu\nu}p_{\nu}| \approx \gamma(F_{\perp}/eE_{cr})$ and $\chi_{\gamma} \approx$ $(\hbar\omega/mc^2)(F_{\perp}/eE_{cr})$, where $F_{\mu\nu}$ is the field-strength tensor, p_{μ} is the particle four-momentum, $\hbar\omega$ is the photon energy, γ is the electron gamma-factor, F_{\perp} is the component of Lorentz force, which is perpendicular to the electron velocity, $E_{cr}=m^2c^3/(e\hbar)=10^{16} \text{ V/cm}$ is the so-called QED characteristic field, \hbar is the Planck constant. χ_e determines photon emission by relativistic electron while χ_{γ} determines interaction of hard photons with electromagnetic field. QED effects are important when $\chi_e \gtrsim 1$ or $\chi_{\gamma} \gtrsim 1$. If $\chi_e \gtrsim 1$ then $\hbar\omega \sim \gamma mc^2$ and the quantum recoil imposed on the electron by the emitted photon is strong. The probability rate of emission of a photon with energy $\hbar\omega$ by relativistic electron with gamma-factor γ can be written in the form [17–19]

$$dW_{em}\left(\xi\right) \; = \; \frac{\alpha mc^2}{\sqrt{3}\pi\hbar\gamma} \left[\left(1 - \xi + \frac{1}{1 - \xi}\right) K_{2/3}(\delta) \right.$$

$$-\int_{\delta}^{\infty} K_{1/3}(s)ds d\xi, \qquad (1)$$

where $\hbar\omega$ is the photon energy, m is the electron mass, c is the speed of light, $\delta = 2\xi/[3(1-\xi)\chi_e]$ and $\xi = \hbar\omega/(\gamma mc^2)$. $\hbar\omega dW_{em}$ can be considered as the energy distribution of the electron radiation power. For electron radiation in constant magnetic field **B** perpendicular to the electron velocity it reduces to the synchrotron radiation spectrum in the classical limit $\chi_e \ll 1$ [18, 20]. The probability rate of electron-positron pair production by decay of a photon with energy $\hbar\omega$ is [17–19]

$$dW_{pair}(\eta_{-}) = \frac{\alpha m^{2} c^{4}}{\sqrt{3}\pi \hbar^{2} \omega} \left[\left(\frac{\eta_{+}}{\eta_{-}} + \frac{\eta_{-}}{\eta_{+}} \right) K_{2/3}(\delta) - \int_{\delta}^{\infty} K_{1/3}(s) ds \right] d\eta, \tag{2}$$

where $\delta = 2/(3\chi_{\gamma}\eta_{-}\eta_{+})$, $\eta_{-} = \gamma mc^{2}/(\hbar\omega)$ and $\eta_{+} = 1 - \eta_{-}$ are the normalized electron and positron energies, respectively. It follows from Eq. (2) that in the classical limit $\chi_{\gamma} \ll 1$ this probability is exponentially small.

To study EPPP dynamics we have developed twodimensional numerical model based on PIC/MC meth-Similar methods has been used previously for modeling of discharges in gases [21]. Recently a onedimensional PIC/MC model has been developed to simulate pair cascades in magnetosphere of neutron stars [22]. However the latter model is electrostatic and the classical approach is used for photon emission with radiation reaction force in the equation of motion. In our numerical model we use more general approach. We exploit the fact that there is a large difference between the photon energy scales in EPPP. The photon energy of the laser and plasma fields is low $(\hbar\omega \ll mc^2)$ while the energy of the photons emitted by accelerated electrons and positrons is very high $(\hbar\omega\gg mc^2)$. The emitted photons are hard and can be treated as particles. Conversely, the evolution of the laser and plasma fields is calculated by numerical solution of the Maxwell equations. Therefore, the dynamics of electrons, positrons and hard photons as well as the evolution of the plasma and laser fields are calculated by PIC technique while emission of hard photons and pair production are calculated by MC method.

The photon emission is modeled as follows. On every time step for each electron and positron we sample a photon emission by a probability distribution which approximates Eq. (1) with the accuracy within 5%. The new photon is included in the simulation region. The coordinates of a new photon are equal to the electron (positron) coordinates at the emission instance. The photon momentum is parallel to the electron (positron) momentum. The electron (positron) momentum value is decreased by the value of the photon momentum. Similar algorithm is used for modeling of pair production by photons. The new electron and positron are added in the simulation region while the photon that produced a pair is removed.

The sum of the electron and positron energy is equal to the photon energy. The pair velocity is directed along the photon velocity at the instance of creation.

The MC part of our numerical model has been benchmarked to simulations performed by other MC codes. We simulated the electromagnetic showers in a static homogeneous magnetic field, the interaction of relativistic electron beam with a strong laser pulse, and the development of electromagnetic cascades in circularly polarized laser pulses. The obtained results are in reasonably good agreement with those published by other authors and are discussed in Ref. [19]. The particle motion and evolution of the electromagnetic field are calculated with standard PIC technique [23]. The PIC part of the model is two-dimensional version of the model used in Ref. [24]. In order to prevent memory overflow during simulation because of the exponential growth of particle number in a cascade, the method of particles merging is used [22]. If the number of the particles becomes too large the randomly selected particles are deleted while the charge, mass, and the energy of the rest particles are increased by the charge, mass, and energy of the deleted particles, respectively.

We use our numerical model to study production and dynamics of EPPP in the field of two colliding linearly polarized laser pulses. The laser pulses have Gaussian envelopes and propagate along the x-axis. The components of the laser field at t=0 are $E_y, B_z=a_0\exp(-y^2/\sigma_r^2)\sin\zeta\left[e^{-(x+x_0)^2/\sigma_x^2}\pm e^{-(x-x_0)^2/\sigma_x^2}\right]$, where the field strengths are normalized to $mc\omega_L/|e|$, the coordinates are normalized to c/ω_L , time is normalized to $1/\omega_L$, $a_0 = |e|E_0/(mc\omega_L)$, E_0 is the electric field amplitude of a single laser pulse, ω_L is the laser pulse cyclic frequency, $2x_0$ is the initial distance between the laser pulses, $\zeta = x - \phi$, and ϕ is the phase shift. The parameters of our simulations are $\phi = 0.8\pi$, $a_0 = 1.2 \cdot 10^3$, $\sigma_x = 125$, $\sigma_r = 40$, $x_0 = \sigma_x/2$ that for the wavelength $\lambda = 2\pi c/\omega_L = 0.8~\mu\mathrm{m}$ correspond to the intensity $3 \cdot 10^{24}$ W/cm², pulse duration 100 fs, the focal spot size 10 μ m at $1/e^2$ intensity level. The cascade is initiated by a single electron located at x = y = 0 with zero initial momentum for t = 0 when the laser pulses approach to each other (the distance between the pulse centers is σ_x).

The later stage $(t=25.5\lambda/c)$ of the cascade development is shown in Fig. 1, where the electron and photon density distributions and the laser intensity distribution are presented. The laser pulses passed through each other by this time instance and the distance between the pulse centers becomes about $1.6\sigma_x$. It is seen from Figs. 1 that the micron-size cluster of overdense EPPP is produced and the laser energy at the backs of the incident laser pulses is spent on EPPP production and heating. The plasma density exceeds the relativistic critical density a_0n_{cr} in about 2 times, where $n_{cr} = m\omega^2/(8\pi e^2)$ is the

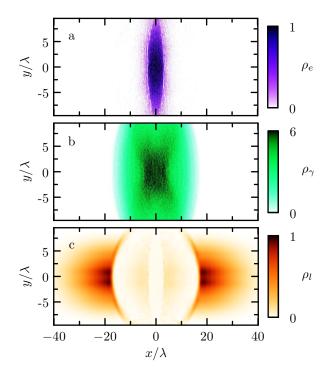


FIG. 1: The normalized electron density $\rho_e = n_e/(a_0 n_{cr})$ (a), the normalized photon density $\rho_\gamma = n_\gamma/(a_0 n_{cr})$ (b) and the laser intensity normalized to the maximum of the initial intensity ρ_l (c) during the collision of two linearly-polarized laser pulses at $t = 25.5 \lambda/c$.

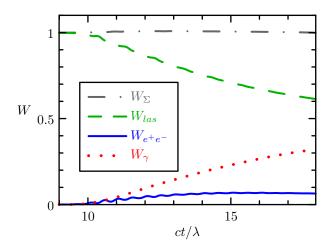


FIG. 2: The electron and positron energy (solid line), the photon energy (dotted line), the laser energy (dashed line) and the total energy of the system (dash-dotted line) as functions of time. All the energies are normalized to the initial energy of the system.

nonrelativistic critical density for the electron-positron plasma. The evolution of the particle and laser energy is shown in Fig. 2. It is seen from Fig. 2 that about a half of the laser energy is absorbed by self-generated EPPP and then mostly reradiated in ultrashort pulse of gamma-quanta. The total energy of the particles in the

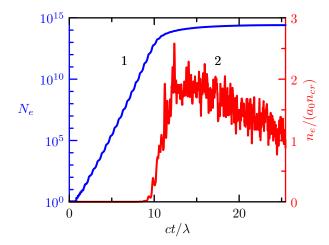


FIG. 3: The number of the electrons produced in the cascade (line 1) and the EPPP density normalized to the relativistic critical density (line 2) as functions of time.

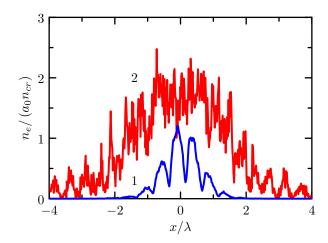


FIG. 4: The profile of the electron density along x-axis at y=0 for initial stage $ct=6.4\lambda$ (line 1) and for the later stage $ct=16.6\lambda$ (line 2) of the cascade development. The electron density for $ct=16.6\lambda$ is normalized to a_0n_{cr} and that for $ct=6.4\lambda$ is normalized to $3.3\times 10^{-6}a_0n_{cr}$.

cascade and the electromagnetic field is conserved with accuracy about 1% during our simulation.

At initial stage of the cascade development the number of created particles is growing exponentially $N \sim e^{\Gamma t}$ [12], where Γ is the multiplication rate. It follows from the energy conservation law that the number of particles that can be created is limited by the laser pulses energy. Thus, at some instant the exponential growth is replaced by much slower growth. Equating the initial energy of laser pulses to the overall particles energy after the pulse collision we get $N \sim a_0^2 \sigma_x \sigma_r^2 N_0 / \bar{\gamma}$, where we assume $N_e \sim N_p \sim N_{ph} \sim N$, N_e , N_p and N_{ph} are the number of electrons, positrons and photons produced by the cascade, respectively, $mc^2\bar{\gamma}$ is the average particle energy, $N_0 = n_{cr}(c/\omega)^3 = \lambda/(16\pi r_e)$, $r_e = e^2/(mc^2)$.

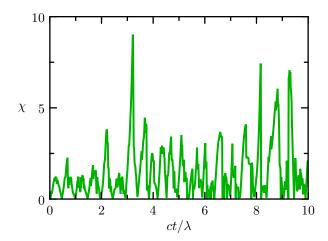


FIG. 5: The dependence of χ_e for the primary electron on time.

The multiplication rate decreases when the field strength goes down, that, by turn, occurs if the plasma density reaches the value a_0n_{cr} . This is in good agreement with the numerical results shown in Fig. 3, where the multiplication rate drops dramatically and EPPP density reaches the value about a_0n_{cr} at the same instant of time $t_s \approx 10\lambda/c$. The value of t_s can be estimated as $t_s \approx \Gamma^{-1} \ln N$. It follows from Fig. 3 that $\Gamma \approx 0.6\omega_L$ for $t < t_s$. The typical lifetime t_{em} for electrons and positrons with respect to hard photon emission can be estimated as $1/\Gamma > 1/\omega_L$ [12]. Thus, for the parameters of numerical simulation $\bar{\gamma}$ can be estimated as $\bar{\gamma} \sim a_0$ hence $N \sim a_0 \sigma_x \sigma_r^2 N_0 \sim 4 \cdot 10^{14}$ and $ct_s/\lambda \sim 9$ that are in good agreement with the corresponding values from Fig. 3.

It is shown in Ref. [15] that the cascades do not arise in B-node of linearly polarized standing electromagnetic wave so far as χ_e and χ_{γ} are less than unity. However, our numerical simulations show that the cascade quasiperiodically develops between B and E nodes of such a wave. This is because under such conditions the electron motion becomes complicated and is not confined to the direction of polarization on the temporal scales about the laser period. It turns out that there occur the time intervals of duration of the order of ω_L^{-1} with $\chi_e > 1$ (see Fig. 5) on which the cascade can develop. The time modulations of $N_e(t)$ and of EPPP density along the xaxis at the initial stage of the cascade development are seen in Fig. 3 and 4, respectively. At the later stages the spatial modulation of the density is strongly smoothed out due to EPPP expansion (see Fig. 4, line 2).

In conclusion we develop the numerical model which allows us to study EPPP dynamics in strong laser field self-consistently. We have demonstrated efficient production of EPPP at the cost of the energy of the laser pulses. We show that even not extremely high intensity laser pulses ($I \sim 10^{24} \ {\rm W/cm^2}$ with duration $\sim 100 \ {\rm fs}$) can produce

overdense EPPP so that the QED effects can be experimentally studied with near coming laser facilities like ELI [6] and HiPER [7]. The simulations and estimates show that for intensity $I > 10^{26} \ {\rm W/cm^2}$ the overdense EPPP can be produced during a single laser period. In such high-intensity regime few-cycle laser pulses can be used in experiments. High-energy photons or electron-positron pair can be also used as a seed to initiate cascade instead of an electron. Photon-initiated cascade can be more suitable for experimental study in low intensity regime $(I \sim 10^{24} \ {\rm W/cm^2})$ because the laser intensity threshold for pair creation in vacuum is about $\sim 10^{26} \ {\rm W/cm^2}$ [25].

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- * Electronic address: kost@appl.sci-nnov.ru
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